CALCULATION OF THE ZONE OF INTENSE RADIAL FRACTURING ACCOMPANYING AN EXPLOSION

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A description of the front of radial cracks, appearing in a brittle elastic body under action of pressure arising at the wall of a cavity inside the body, was proposed in [1, 2]. Later, a model taking into account other forms of failure as well was formulated in [3]. Based on this model, calculations of the mechanical action of an explosion in brittle rock were carried out in [4-6]. In this paper, as in [1, 2, 5], we concentrate on the zone of intense radial fracture.

Experimental data [7] show that at the initial stage the velocity of the radial crack front is determined by the propagation of some phase of the stretching azimuthal stresses, while the fracture zone consists of fine separate cracks. It is this particular stage that is described by the model in [1-3], according to which the following criterion must be satisfied at the leading edge of the fracture front:

$$\sigma_\varphi = \sigma_*, \tag{1}$$

where σ_* is the tensile strength of the rock.

A characteristic of this model is the fact that the velocity of the fracture front drops continuously from an initial value to zero. However, according to experimental data [7, 8], the velocity of the front, dropping to a value equal approximately to the velocity of Rayleigh waves, remains for a long period of time practically constant and stretching stresses in the wave, apparently, weaken to such an extent that they are no longer strong enough to generate new cracks. This means that condition (1) at the stage of development of the explosion indicated is not satisfied. In connection with this, it was proposed in [9] that (1) be replaced by the condition that the velocity of the fracture front R' be replaced by the limiting velocity of cracks v_0 relative to the material

$$R' - v = v_0, \tag{2}$$

where v is the value of the mass velocity in front of the fracture front.

Let us examine the following model problem. In an infinite medium, loaded by hydrostatic pressure p_h, there is a spherical cavity with radius a_0. Initially, a pressure p_0 arises in the cavity. The medium is assumed to be elastobrittle, and we shall neglect its compressibility. For a sufficiently high pressure p_0, fracture begins in the mass surrounding the cavity. We shall examine fracture as a result of the formation of radial cracks, arising under the action of stretching azimuthal stresses σ_φ. There exists a range of values of the pressure p_0, for which spalling fracture does not occur. We shall assume that

$$p_0 \ll \rho c^2, \tag{3}$$

where ρ and c are, respectively, the density and velocity of transverse waves in the unfractured rock. In this case, it is permissible to assume that the fractured medium is also incompressible and has the same density ρ.

The medium contains natural fracturing, i.e., it contains seed microcracks. The concentration of microcracks and their size are assumed to be such that the distance between the cracks d_0 and their size l_0 is much less than the size of the cavity a_0. If the stresses attain the tensile strength σ_* or shear strength τ_*, then the microcracks begin to grow and a zone with fractured material forms.

Until the stress in the medium attains critical values, fracture does not begin and, therefore, we have a Sharp problem. We shall present the equations describing the problem at this stage (the medium is incompressible)

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$$\rho\partial^2u/\partial t^2 = \partial\sigma_r/\partial r + 2(\sigma_r - \sigma_\varphi)/r,$$
$$\partial u/\partial r + 2u/r = 0,\ \sigma_r - \sigma_\varphi = 2\rho c^2(\partial u/\partial r - u/r), \tag{4}$$

where u is the radial component of the displacement vector. To system (4) it is necessary to add zero initial conditions and boundary conditions at the wall of the cavity

$$\sigma_r(a_0,\ t) = -p_0. \tag{5}$$

The solution of system (4) has the form

$$v = F'(t)/r^2,\ \sigma_r = -\rho F''(t)/r - 4\rho c^2F(t)/r^3 - p_h,$$
$$\sigma_\varphi = -\rho F''(t)/r + 2\rho c^2F(t)/r^3 - p_h, \tag{6}$$

where v is the mass velocity. In determining the function F(t) from the boundary condition (5), we obtain an ordinary differential equation

$$p_0 - p_h = \rho F''/a_0 + 4\rho c^2F/a_0^3. \tag{7}$$

The primes indicate differentiation with respect to time. Equation (7) must be supplemented with zero initial conditions, after which the form of the function F(t) is easily determined:

$$F(t) = a_0^3\,\frac{p_0 - p_h}{4\rho c^2}\left[1 - \cos\left(2\frac{ct}{a_0}\right)\right]. \tag{8}$$

Equations (4)-(8) are valid until fracture begins at the boundary of the cavity:

$$\sigma_\varphi(a_0, t) - \sigma_r(a_0\, t) = 2\tau_* \tag{9}$$

or

$$\sigma_\varphi(a_0, t) = \sigma_*. \tag{10}$$

Depending on which of the two conditions (9) or (10) is satisfied in time first, shear or tensile fracture begins at the cavity. Using the relations presented above, it can be shown that in the case $\tau_* < (3/4)\sigma_*$, fragmentation begins under the condition $p_0 > (2/3)\tau_* + p_H$ in the cavity at some time determined from (9). At lower pressure p_0, fracture does not occur and the problem remains purely elastic. When the reverse inequality is satisfied

$$\tau_* > (3/4)\,\sigma_* \tag{11}$$

it can be shown that in the region of initial pressures in the cavity

$$(1/2)\,\sigma_* + (3/2)\,p_h < p_0 < 2\tau_* - \sigma_* \tag{12}$$

radial cracks begin to grow away from the cavity at some time.

Shear failure in this case does not occur; it begins at higher pressures in the cavity. Concentrating only on the propagation of the radial fracture front, we shall assume that conditions (11) and (12) are satisfied. In this case, after the stage of purely elastic expansion of the cavity, a zone with radial cracks forms around it.

At the initial time of fracture t_0, determined from (10), we have from (6) and (8)

$$F(t_0) = \varphi_0,\ F'(t_0) = \varphi_1, \varphi_0 = \frac{a_0^3}{6}\,\frac{p_0 + \sigma_*}{\rho c^2},$$
$$\varphi_1 = \frac{1}{6}\,\frac{a_0^2}{\rho c}\sqrt{9\,(p_0 - p_h)^2 - (p_0 - 2\sigma_* - 3p_h)^2}. \tag{13}$$

Expressions (13) are initial conditions for the solution of the problem at the fracture stage.

As noted previously, it may be assumed that the density of the rock does not change with fracturing. For this reason, in order to describe the fractured region, Eqs. (4), in which the last equation (Hooke's law) must be replaced by the condition [1]

$$\sigma_\varphi = 0. \tag{14}$$

are applicable. The solution of Eq. (14), satisfying the boundary condition (5), has the form

$$v = a_0^2 b'(t)/r^2,\ \sigma_r = \rho a_0^2 b''(t)/r - \rho a_0^3 b''(t)/r^2 - p_0 a_0^2/r^2. \tag{15}$$

Equations (15) describe the solution in the fracture zone. In the unfractured zone, as before, Eqs. (4) are used and, therefore, the solution in the form (6). We shall denote the unknown function F(t) entering into (6) as f(t). The problem will be solved if it is possible to determine the functions b(t), f(t) and the size of the zone of intense radial fracturing R(t).

In order to determine the three unknown functions b(t), f(t), R(t), it is necessary to write three equations: three conditions for joining solutions at the fracture front. For the first two equations, we shall choose the laws of conservation of mass and momentum. Assuming that the density of the medium does not change with fracturing, and using Eqs. (6) and (7) as well, these two equations can be put into the form

$$a_0^2 b(t) = f(t),\ \rho(2R^2 - a_0 R) f'' + 4\rho c^2 f = p_0 a_0^2 R - p_h R^3. \tag{16}$$

If the dependence of the radius of the fracture front R on time is known, then the functions b(t) and f(t) are determined from (16). In the case $R = a_0$ (fracture has not begun), Eqs. (16) go over into (7). In the case $R = \text{const} > a_0$ (fracture has ceased), it follows from (16) that

$$f(t) = R\frac{p_0 a_0^2 - p_h R^2}{4\rho c^2} + C_1 \cos\left(\frac{2ct}{\sqrt{2R^2 - a_0 R}}\right) + C_2 \sin\left(\frac{2ct}{\sqrt{2R^2 - a_0 R}}\right), \tag{17}$$

where C_1, C_2 are arbitrary constants. Comparing (17) and (8), it is evident that in the presence of the fracture zone, the period of oscillations of particles in the medium increases.

In general, when the form of the function R(t) is not known, the system (19) must be supplemented by one more equation: the fracture criterion. Let us assume that the functions f(t) and R(t) can be represented as series in some vicinity of the point t_0 (time of initial fracture):

$$R(t) = \sum_{n=0}^{\infty} A_n (t - t_0)^n,\ f(t) = \sum_{n=0}^{\infty} D_n (t - t_0)^n, \tag{18}$$

where $A_0 = a_0$. In the case of the fracture criteria used in what follows, the validity of the expansions (18) can be proved using Cauchy's theorem for ordinary differential equations with a holomorphic right side.

From the initial conditions (13), it follows that $D_0 = \varphi_0$, $D_1 = \varphi_1$. Using the representation (18), we obtain from (6) and (16) to within terms of order $(t - t_0)^2$

$$\sigma_\varphi(R, t) \simeq \sigma_* + (t - t_0)\left\{6\rho c^2 \varphi_1 / a_0^2 - V\left(\frac{2}{3} p_0 + p_h + \frac{11}{3}\sigma_*\right)\right\}\Big/ a_0, \tag{19}$$

where $V = A_1$ is the initial value of the velocity of the fracture front. It is evident from (19) that the higher the velocity of the fracture front V, the more rapidly the azimuthal voltages at leading edge of the front $\sigma_\varphi(R, t)$ decrease with time. This assertion is valid not only for times close to the onset of fracture, but for other times as well.

The formation of a zone of intense radial fracturing is related to two fracture mechanisms. First, multiple quasi-three-dimensional crack formation can occur on the fracture front. In this case, the azimuthal stretching stresses at the leading edge of the front must be less than the threshold stresses σ_*, while the velocity of the front R'(t) must not be less than the velocity of growth of a single crack v_0. A simple and natural criterion for fracture in this case is relation (1). It should be emphasized that this criterion is valid only in the case $R'(t) > v_0$. Second, when the velocity of the fracture front decreases to the value v_0, processes related to the growth of a system of interacting mainline cracks become important.

In order to clarify the nature of the interaction of cracks, we shall examine a two-dimensional model problem. Assume that we have an infinite system of parallel cracks of length $2l$ in the xy plane, situated symmetrically relative to the y axis separated by a distance of 2d. The cracks are located in an external homogeneous field of stretching stresses σ_0. It is necessary to determine the motion of such a system of cracks. This problem was examined in [10], where it was shown that the motion of cracks is unstable. Let every other crack have an identical increment to its length. Then, the velocity of the longer cracks increases, while the velocity of the shorter cracks decreases until they are no longer separated. A new system of cracks separated by distance 4d is formed, for which the same procedure can be carried out. Thus, with time, the distance between cracks increases, while their velocity increases. These results, obtained in [10], are related to the interaction of cracks, which is important for $d \ll l$.

In a real medium, there is some distribution of cracks with respect to length. Then, as is evident from the results presented above, the velocity of the longest cracks, forming the fracture front, rapidly increases to a limiting value v_0, while the shorter cracks stop, i.e., the velocity of the fracture front can be assumed to be constant:

$$l'(t) = R'(t) = v_0, \tag{20}$$

while the number of growing cracks decreases.

It is known that for the continuation of a crack, the stress approaches infinity according to the law $\sigma_y = K/\sqrt{2\Delta}$, , where Δ is the distance from the tip of the crack; I is the intensity coefficient, which is determined from a solution of the appropriate elastic problem. The condition for growth of a system of cracks described above can be written in the form

$$\sigma_0 \sqrt{d}\,\xi(l') = \text{const}, \tag{21}$$

where the quantity $\sigma_0\sqrt{d}$ up to a numerical factor coincides with the static value of the stress intensity coefficient in the case $d \ll l$, while the factor $\xi(l')$ takes into account the dynamic effects, arising with the motion of cracks. Taking into account (20), (21) can be represented in the form $\sigma_0\sqrt{d}$ = const. In application to the problem of the expansion of a cavity, we shall write this condition as

$$d/d_0 = (\sigma_*/\sigma_\varphi)^2, \tag{22}$$

where σ_φ is the value of the azimuthal stresses at the leading edge of the fracture front; d_0 is the initial distance between cracks (parameter of natural fracturing of rock).

Thus, for propagation of a fracture front formed by a system of main line cracks, we shall use the criterion (20), which on the strength of (3) coincides with criterion (2) proposed in [9]. In this case, the distance between cracks (and, therefore, the number of growing cracks) is a variable quantity and is determined from (22).

The results presented above are strongly related to the interaction of cracks, i.e., to the condition $d \ll l$. Since it is clear a priori that the size of the zone of intense radial fracturing, on the strength of (12), will be of the order of the cavity sizes, for simplicity, we shall take the condition for applicability of all the equations presented above in the form

$$d \ll a_0. \tag{23}$$

Further propagation of the fracture zone (and actually a small number of noninteracting cracks) falls outside the scope of the proposed model and will not be examined.

Depending on the parameters of the problem, the following cases are possible. For gas pressures in the cavity $p_0 < (1/2)\sigma_* + (2/2)p_h$ (the region I in Fig. 1), the problem becomes purely elastic. For pressure in the cavity $p_0 > 2\tau_* - \sigma_*$ (region IV in Fig. 1), in the vicinity of a crack, shear fracture begins. In the intermediate pressure range p_0, the fracture zone is formed by radial cracks. We shall introduce the quantity p_*, determined from the equation

$$\sqrt{9(p_* - p_h)^2 - (p_* - 3p_h - 2\sigma_*)^2} = v_0((2/3)\,p_* + p_h + (11/3)\,\sigma_*)/\text{sec}\,. \tag{24}$$

It can be shown that in the case $p_* < p_0 < 2\tau_* - \sigma_*$ (region III in Fig. 1), the fracture on the cavity begins according to criterion (1), and then, when the velocity of the fracture front drops to the value v_0, the propagation of the fracture front is described by criterion (20). In the case $(1/2)\sigma_* + (3/2)p_h < p_0 < p_*$ (region II in Fig. 1), fracture from the very beginning is determined by criterion (20).

The fracture criterion (1) or (20) and Eq. (16) form a complete system for determining the unknown functions b(t), f(t), R(t). In the case of volume fracturing (1), the problem reduces to a numerical solution of ordinary differential equations. For growth of a system

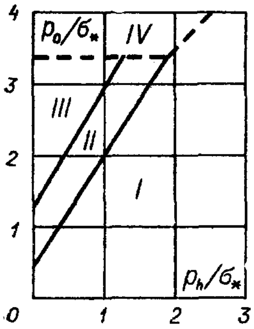

Fig. 1

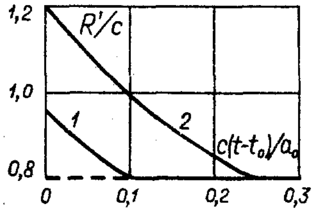

Fig. 2

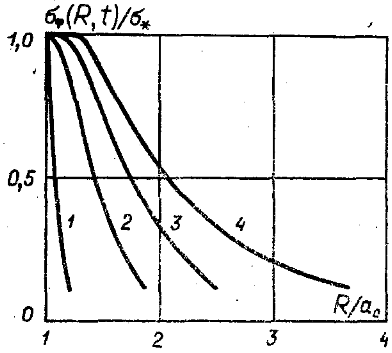

Fig. 3

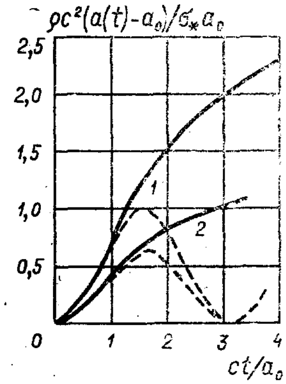

Fig. 4

of main line cracks $R = R_0 + v_0(t - t_0)$, we obtain from (16) for the function $f(t)$ a differential equation, whose solution is expressed in terms of the hypergeometric function. The results of the calculations are presented in Figs. 2-6. In all variants of the calculation, the following values of the parameters were taken: $v_0 = 0.8$ sec, $a_0 = 100d_0$. The calculation was terminated when condition (23) $d = a_0$ was satisfied.

Figure 2 presents the dependence of the velocity of the fracture front on time for initial pressures $p_0 = 1.5\sigma_*$ and $p_0 = 2\sigma_*$ (curves 1 and 2, respectively) in the cavity. The back pressure in both cases was assumed to equal zero. The dashed line corresponds to pressures in the cavity $p_0 \leqslant p_*$, where p_* is defined in (24).

Figure 3 presents the azimuthal stresses $\sigma_\varphi(R, t)/\sigma_*$ at the leading edge of the fracture front as a function of the size of the fracture zone R/a_0 with pressures in the cavity $p_0 = 2\sigma_*$. Curves 1 and 2 correspond to the back pressure $p_h = \sigma_*$ and $p_h = 0.5\sigma_*$, while curves 3 and 4 to the back pressure $p_h = 0.2\sigma_*$ and $p_h = 0$. It is evident that the stage of quasivolume fracturing (curves 3 and 4) takes up an insignificant part of the overall fracture time.

Figure 4 shows the dependence of the cavity radius on time. Curves 1 and 2 correspond to pressures in the cavity $p_0 = 2\sigma_*$ and $p_0 = 1.2\sigma_*$. The back pressure in both cases equals zero. The dashed lines are the same curves, but obtained by solving a purely elastic problem.

Curves 1 and 2 in Fig. 5 give the finite radius of the zone of intense radial fracturing as a function of pressure p_0/σ_* with lithostatic pressure $p_h = 0$ and $p_h = 0.25\sigma_*$, correspondingly, at the condition for terminating the calculation (23).

Figure 6 shows the average distance between cracks d/d_0 as a function of the coordinate r/a_0. Curves 1-3 correspond to pressure in the cavity $p_0 = 0.5\sigma_*$, $p_0 = \sigma_*$ and $p_0 = 2\sigma_*$. The back pressure in all three cases equals zero.

Thus, it is shown in this paper that with the formation of a zone of intense radial fracturing, two fracture mechanisms play an important role: quasivolume, in which multiple crack formation occurs on the fracture front, and growth of a system of mainline cracks with which new cracks are not formed on the fracture front. It is shown for the solution of a model problem of the action of a spherically symmetrical explosion how one of the fracture mechanisms transforms into the other. The dependences of oscillations of the particles in the medium in

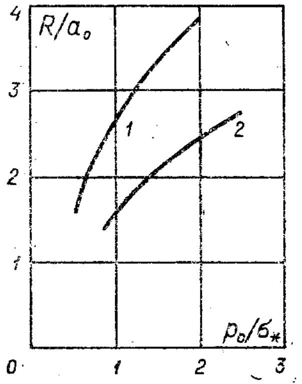

Fig. 5

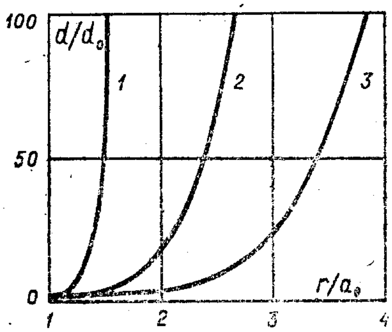

Fig. 6

the mass surrounding the cavity and the dependence of the average size of a piece on radius are obtained. The dependence of the fracture front velocity on time coincides qualitatively with the experimental dependence.

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